Upper Bounds on Coarsening Rates

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This talk presents joint work with Felix Otto, proving rigorous upper bounds on the coarsening rates for two standard models of surface-energy-driven interfacial dynamics.

The models we address are the Mullins-Sekerka law and motion by surface diffusion. Each preserves volume and decreases perimeter. Each has, moreover, a scale invariance, making it natural to guess that the associated coarsening is self-similar. If so then the typical length scale $L(t)$ would behave as $L(t)\ t^{1/3}$ for Mullins-Sekerka and $L(t)\ t^{1/4}$ for surface diffusion. Such behavior is seen in stochastic simulations but has never been demonstrated with mathematical rigor.

The proposed scaling law can be decomposed into two rather different assertions: (a) an upper bound for $L(t)$, showing that microstructure cannot coarsen faster than the similarity law; and (b) a lower bound for $L(t)$, showing that the microstructure must coarsen at least as fast as the similarity law. Assertion (b) is subtle – it can only be true generically or with probability 1 – and it remains open. Assertion (a) however is different and easier: its essential content is kinematic, and it should be true universally.

Our main achievement is to prove a weak version of assertion (a). The argument uses only the basic energy relations, interpolation inequalities, and a simple ODE argument.