# Transmission-Stefan Problems Arising in Czochralski Process of Crystal Growth

Nobuyuki Kenmochi

The Czochralski pulling method is now widely used in order to produce single crystals from melt substances. From the mathematical point of view it is composed of the following three questions (P1), (P2) and (P3):

(P1) By pulling the seed crystal, the single crystal grows below the seed crystal. Therefore, the material region (of the melt and crystal moves smoothly in time. How are we determine this time dependent material region?

(P2) By the movement of the material region a flow of substance and heat convection is caused in it. It is also a difficult question how to deternine the convective vector field in the time dependent material domain.

(P3) The solid-liquid phase transition occurs in the time dependent material domain and our question is how to determine the free boundary (solid-liquid interface). This is nothing but a two-phase Stefan problem with convection in a moving domain.

In this talk, we pay our attension only to (P3), assuming that the moving material domain is prescribed as well as the convective vector field in it. More precisely we propose two Stefan problems in time-dependent domain. One is a two-phase Stefan problem (the enthalpy formulation) with prescribed convective vector field in a timedependent domain. We give an existence and uniqueness result for it and note that the convective vector field plays an important role in its weak variational formulation. Another is a transmission problem of a two-phase Stefan problem in a moving subdomain of a fixed domain and a heat equation in the complement of the the moving domain; namely on the common boundary the continuity of temperature and of heat flux is required as the transmission condition. We give an existence result for this problem.

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