Shape deformations and analytic continuation in free boundary problems

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In this talk I will present a general approach to the analysis of steady states and nonlinear stability for a class of free-boundary problems. The approach is based on the explicit consideration of series solutions in a parameter measuring variations of domains (e.g. steady state or initial configurations) from separable geometries (e.g. planes, spheres, etc). The method relies on the derivation of formulas for the recursive evaluation of (Taylor) approximations of arbitrary order in the variation parameters, and on the iterative estimation of the growth, in appropriately defined spaces, of the resulting functional coefficients. We shall present a variety of instances where this procedure leads to general results on analyticity of solutions, on existence of complex steady states and on nonlinear stability of equilibria. Particular attention will be given to examples of "curvature driven" free-boundary problems, including the classical Stefan problems, models of capillary fluid drops, water waves, tumor growth, etc. We shall further show that the relevance of our studies goes beyond the theoretical, as it uncovered the mechanism behind the observed performance of a class of numerical algorithms, based on shape-perturbation theory, that have been used in these and other contexts. Indeed, our research shows that the standard implementations of these schemes can suffer from severe ill-conditioning as a result of pronounced cancellations in the underlying recursions. Moreover, our work further suggests alternative implementations with greatly improved properties of numerical stability and convergence, enhanced by methods of analytic continuation. As we shall demonstrate these modifications can have a dramatic effect on the accuracy and applicability of perturbative numerical approaches to (boundary value and) free-boundary problems.