Magnetization modes and spin-wave instabilities in nonlinear magnetization dynamics

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The Landau-Lifshitz-Gilbert (LLG) equation is broadly used to describe nonlinear magnetization dynamics in micromagnetic systems. In its more general form, the LLG equation appears as a nonlinear partial differential equation for the magnetization vector field, coupled with Maxwell equations. Its mathematical structure and the physical consequences of its use in micromagnetics have not yet been fully explored and understood. In this talk, analytical results about LLG dynamics are obtained for problems characterized by uniaxial rotational invariance. Exact analytical solutions of the LLG equation are derived for the case of nonlinear large motions of the magnetization in a body with uniaxial symmetry, subject to a circularly polarized external magnetic field. These solutions represent pure time-harmonic, spatially uniform magnetization modes, with no generation of higher-order harmonics, despite the strongly nonlinear nature of the problem. The conditions under which no uniform-mode solution is stable and quasi-periodic magnetization modes, with spontaneous breaking of the rotational symmetry, necessarily appear in the dynamics, are discussed. The stability of the nonlinear uniform-mode solutions can be analytically studied by introducing generalized spin-waves, intended as thermally-generated plane-wave perturbations of the magnetization mode. The equation obeyed by these generalized spin-waves, obtained by linearizing the LLG equation around the analytically known uniform-mode solution, is an equation with periodic coefficients. It is shown that there exists a direct connection between Floquet theory for this class of equations and the determination of the dispersion relation characterizing the generalized spin-waves. Conditions for the onset of parametric spin-wave instabilities are derived for arbitrary values of the amplitude and frequency of the external magnetic field. It is shown that stability of the magnetization modes depends on their preparation history. These predictions are discussed in relation to the problem of ferromagnetic resonance and that of magnetization switching in ferromagnetic media.