Penalty method and problems of liquid - solid interaction

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The subject of this talk is concerned with existence results for a coupled system of non-linear partial and ordinary differential equations modelling the motion of several rigid bodies inside of a fluid. The governing equations for the fluid flow are the classical Navier-Stokes system, whereas the motion of the rigid bodies is governed by the balance equations for linear and angular momentums. Problems of this kind arise in various mechanical situations. In particular, we should be able to solve them, if we study phase transitions of the liquid - solid type and want to take into account the motion of the substance. The mathematical treatment of the problem assumes overcoming the following main difficulties. Because the bodies change their positions with time, we have a problem for the Navier-Stokes equations in a varying domain. In addition, the motion of the bodies is not known a priory so that we have a free-boundary problem. The most considerable difficulty is related to possible collisions of the bodies. An approach to the problem consists in applying the penalty method. The bodies are considered as fluid domains where the viscosity tends to infinity. This technique allowed us to obtain global solvability results. Moreover, some qualitative facts which plays an essential role in proving the solvability were established. For example, if the bodies have sufficiently smooth boundaries, they can touch one other only with zero relative velocity. In addition to the review of known issues, we present recent results relating to the solvability of the above discussed problem concerned with the self-propelled motion of bodies in a fluid.